Gravity-driven thin film flow over topography: internal flow and associated free surface disturbance

Sergii Veremieiev[1] and Philip H Gaskell[1]

[1] *School of Engineering and Computing Sciences, Durham University, Durham, DH1 3LE, UK*
Corresponding author: p.h.gaskell@durham.ac.uk

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Solution of the full Navier-Stokes (N-S) equations for the problem of gravity-driven thin film flow over surface features overcomes many of the restrictions, other than physically realisable ones, associated with simpler models based on the long-wave approximation [1], such as choice of capillary number, film thickness or a feature's aspect ratio. The internal velocity field forms part of the solution enabling the flow topology to be explored as the surface geometry, Reynolds number, Re, and capillary number, Ca, are varied. The problem investigated is that of a continuous thin fluid film flowing down an inclined rigid substrate containing a trench-like feature; the fluid involved has constant density, viscosity and surface tension. Two- and three-dimensional solutions are obtained using a Bubnov-Galerkin mixed-interpolation finite element scheme, free-surface parametrisation based on the Arbitrary Lagrangian-Eulerian method of spines and a direct parallel multifrontal method. The three-dimensional solutions represent the first of their kind, enabling both the internal flow topology and free-surface disturbance generated to be explored simultaneously. They provide a valueable benchmark with which to assess the limitations of models based on the long-wave approximation.

Comparison of solutions for the free-surface disturbance generated with their counterparts obtained using the depth-averaged model of [2], reveals both sets of results to be in good agreement. The difference between them is more apparent as Ca, or the ratio of trench depth to length is increased, the effect of which is exacerbated further the higher the value of the associated Re. Analysing the internal flow topology, reveals that across the trench the three-dimensional solution is topologically different internally from its two-dimensional counterpart [3]: whereas for the latter eddies form closed elliptic trajectories, for the former, at the mid-plane, they may instead be foci or nodes (not closed logarithmic spiral or power law trajectories); limit cycles are possible as well, [4].

In addition, the three-dimensional flow case exhibits different flow topologies, dependent not only on the trench geometry in both the streamwise and spanwise directions but also on Ca and Re. For Ca = 0.001 and Re = 0 the flow trajectories inside the trench form closed eddies; for Re = 10, the trajectories are no longer closed but give rise to a stable focus – instead they encroach into the trench near the symmetry mid-plane swirl around several times, during which they are displaced laterally away from the mid-plane and exit the trench close to its side before exiting and travelling downstream; similar features have been reported for flow in a three-dimensional lid-driven cavity, see [5]. In contrast, increasing Ca to 1.0 when Re = 0, the critical point in the mid-plane of the trench is an unstable focus, the trajectories swirl around but out of the eddy exiting the trench near the symmetry mid-plane. This is typical of the case for film thickness where the size of the free surface disturbance generated impacts strongly on the internal flow topology in the vicinity of the trench, independent of inertia effects. The flow patterns that can exist reveal a rich set of features; for example, for a sufficiently small ratio of trench depth to length, when the corner eddies present are small compared to the length of the trench, the internal velocity profile across the film becomes effectively self-similar parabolic over the entire flow domain.

References

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