Gradient flow perspective on thin-film bilayer flows

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We study gradient flow formulations of thin-film bilayer flows with triple-junctions between liquid/liquid/air phase, which were actively investigated in the recent decade, see e.g. [1]-[3] and references therein.

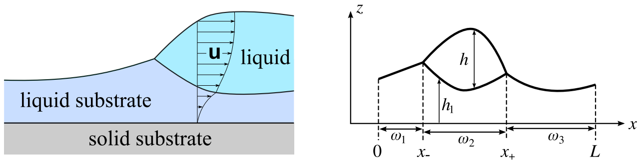

Fig. 1: Liquid bilayer flow on a solid substrate.

First we highlight the gradient structure in the Stokes free-boundary flow and identify its solutions with the well-known PDE with boundary conditions. Next we propose a similar gradient formulation for the corresponding reduced thin-film model and formally identify solutions with those of a PDE problem suggested previously in [1]. A robust numerical algorithm for the thin-film gradient flow structure is then provided in the nature similar to one in [6] and based on the associated Rayleigh-Onsager variational principle. Using this algorithm we compare the sharp triple-junction model with precursor models. For their stationary solutions a rigorous connection is established using Γ-convergence.

For time-dependent solutions the comparison of numerical solutions shows a good agreement for small and moderate times. Finally we study spreading in the zero-contact angle case, where we compare numerical solutions with asymptotically exact source-type solutions.

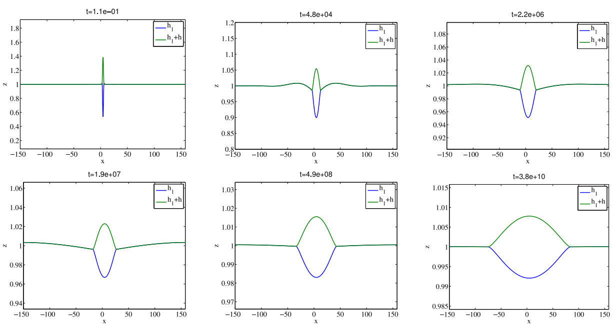

Fig. 2: Spreading of drop converging to a source-type solution.

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