Lubrication theory for thixotropic fluids

Stephen K. Wilson[1], David Pritchard[1] and Catriona R. McArdle[1]

[1] *Department of Mathematics and Statistics, University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH, UK*
Corresponding author: s.k.wilson@strath.ac.uk

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Thixotropy can be defined as time-dependent fluid behaviour in which viscosity decreases as the structure of the fluid is broken down when it is sheared, but recovers to its original value when the shearing is removed. Crucially, neither the build-up nor the breakdown of the structure is, in general, instantaneous, leading to history-dependent rheology (see, for example, [1,2]). Thixotropic behaviour is observed in many fluids used in coating (such as inks and paints), as well as in many other physical contexts (such as muds and waxy crude oils) and so there is considerable interest in developing and analysing theoretical models for the flow of such fluids.

In the simplest models for thixotropy, the state of the microstructure is described by a phenomenological structure parameter, λ. The evolution equation for λ is

$$\hat{u}\frac{\partial \lambda}{\partial \hat{x}} + \hat{v}\frac{\partial \lambda}{\partial \hat{y}} = \hat{f}(\hat{\Gamma}, \lambda) \tag{1}$$

and represents both advection of the microstructure and its build-up and breakdown. To represent the latter, which are assumed to depend on the total shear rate, $\hat{\Gamma}$, we use the rather general structure evolution model of Mewis and Wagner [2], namely

$$\hat{f}(\hat{\Gamma}, \lambda) = -\hat{k}_1\hat{\Gamma}^{a/2}\lambda^b + \hat{k}_2\hat{\Gamma}^{c/2}(1-\lambda)^d, \tag{2}$$

where $\hat{k}_1$ and $\hat{k}_2$ are the breakdown and build-up rates, respectively, while a, b, c and d are non-negative exponents.

Although we develop the lubrication framework in some generality, we adopt the simple constitutive law of Moore [3], namely $\hat{\eta} = \hat{\eta}_0\lambda$, where $\hat{\eta}$ is the apparent viscosity and $\hat{\eta}_0$ is a viscosity parameter.

We describe a general formulation of the governing equations for the slow, steady flow of a thixotropic or antithixotropic fluid.

We demonstrate how the lubrication equations can be further simplified in the weakly advective regime in which the advective Deborah number, $\mathcal{D}$, is comparable to the aspect ratio of the flow, δ ($\ll 1$), and present illustrative analytical and semi-analytical solutions for particular choices of the constitutive and kinetic laws, including a purely viscous Moore–Mewis–Wagner model.

The lubrication results also allow the calibration and validation of cross-sectionally averaged, or otherwise reduced, descriptions of thixotropic flow, and we employ them to explain why such descriptions may be inadequate.

Finally, we indicate directions for future work, including the strongly advective regime and unsteady flows, the latter building on our previous work on the Stokes problem for a thixotropic fluid [4].

References

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